Chapter 12: Equations

Equation 12.1:

$$b_1 = \frac{\sum_{i=1}^{n}\left(\left(x_{1i}-\overline{x}_1\right)-\frac{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)\left(x_{1i}-\overline{x}_1\right)}{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)^2}\left(x_{2i}-\overline{x}_2\right)\right)y_i}{\sum_{i=1}^{n}\left(x_{1i}-\overline{x}_1\right)^2-\frac{\left(\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)\left(x_{1i}-\overline{x}_1\right)\right)^2}{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)^2}}$$

Equation 12.2:

$$\mathrm{V}\left(b_1\right) = \mathrm{V}\left(\frac{\sum_{i=1}^{n}\left(\left(x_{1i}-\overline{x}_1\right)-\frac{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)\left(x_{1i}-\overline{x}_1\right)}{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)^2}\left(x_{2i}-\overline{x}_2\right)\right)y_i}{\sum_{i=1}^{n}\left(x_{1i}-\overline{x}_1\right)^2-\frac{\left(\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)\left(x_{1i}-\overline{x}_1\right)\right)^2}{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)^2}}\right)$$

Equation 12.3:

$$\mathrm{V}\left(b_1\right) = \frac{\sum_{i=1}^{n}\left(\left(x_{1i}-\overline{x}_1\right)-\frac{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)\left(x_{1i}-\overline{x}_1\right)}{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)^2}\left(x_{2i}-\overline{x}_2\right)\right)^2 \mathrm{V}\left(y_i\right)}{\left(\sum_{i=1}^{n}\left(x_{1i}-\overline{x}_1\right)^2-\frac{\left(\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)\left(x_{1i}-\overline{x}_1\right)\right)^2}{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)^2}\right)^2}$$

Equation 12.4:

$$V\left(b_1\right) = \frac{\sigma^2 \sum_{i=1}^{n} \left(\left(x_{1i} - \overline{x}_1\right) - \frac{\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)\left(x_{1i} - \overline{x}_1\right)}{\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)^2} \left(x_{2i} - \overline{x}_2\right) \right)^2}{\left(\sum_{i=1}^{n} \left(x_{1i} - \overline{x}_1\right)^2 - \frac{\left(\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)\left(x_{1i} - \overline{x}_1\right) \right)^2}{\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)^2} \right)^2}$$

Equation 12.5:

$$\sum_{i=1}^{n} \left(x_{1i} - \overline{x}_1\right)^2 - \sum_{i=1}^{n} \left(2 \frac{\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)\left(x_{1i} - \overline{x}_1\right)}{\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)^2} \left(x_{1i} - \overline{x}_1\right)\left(x_{2i} - \overline{x}_2\right) \right)$$

$$+ \sum_{i=1}^{n} \left(\left(\frac{\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)\left(x_{1i} - \overline{x}_1\right)}{\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)^2} \right)^2 \left(x_{2i} - \overline{x}_2\right)^2 \right)$$

Equation 12.6:

$$\sum_{i=1}^{n} \left(x_{1i} - \overline{x}_1\right)^2 - \frac{\left(\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)\left(x_{1i} - \overline{x}_1\right) \right)^2}{\sum_{i=1}^{n} \left(x_{2i} - \overline{x}_2\right)^2}$$

Equation 12.7:

$$\mathrm{V}(b_1)=\frac{\sigma^2}{\sum_{i=1}^{n}(x_{1i}-\overline{x}_1)^2-\frac{\left(\sum_{i=1}^{n}(x_{2i}-\overline{x}_2)(x_{1i}-\overline{x}_1)\right)^2}{\sum_{i=1}^{n}(x_{2i}-\overline{x}_2)^2}}$$

Equation 12.8:

$$\mathrm{V}(b_2)=\frac{\sigma^2}{\sum_{i=1}^{n}(x_{2i}-\overline{x}_2)^2-\frac{\left(\sum_{i=1}^{n}(x_{2i}-\overline{x}_2)(x_{1i}-\overline{x}_1)\right)^2}{\sum_{i=1}^{n}(x_{1i}-\overline{x}_1)^2}}$$

Equation 12.9:

$$\sum_{i=1}^{n}(x_{1i}-\overline{x}_1)^2=\sum_{i=1}^{n}e^2_{(x_1x_2)i}+b^2_{x_1x_2}\sum_{i=1}^{n}(x_{2i}-\overline{x}_2)^2$$

Equation 12.10:

$$\sum_{i=1}^{n}(x_{1i}-\overline{x}_1)(x_{2i}-\overline{x}_2)$$

Equation 12.11:

$$\mathrm{V}(b_1)=\frac{\sigma^2}{\sum_{i=1}^{n}e^2_{(x_1x_2)i}}$$

Equation 12.12:

$$\mathrm{V}\left(b_1\right)=\frac{\sigma^2}{\left(1-\frac{\left(\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)\left(x_{1i}-\overline{x}_1\right)\right)^2}{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)^2\sum_{i=1}^{n}\left(x_{1i}-\overline{x}_1\right)^2}\right)\sum_{i=1}^{n}\left(x_{1i}-\overline{x}_1\right)^2}$$

Equation 12.13:

$$\frac{\left(\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)\left(x_{1i}-\overline{x}_1\right)\right)^2}{\sum_{i=1}^{n}\left(x_{2i}-\overline{x}_2\right)^2\sum_{i=1}^{n}\left(x_{1i}-\overline{x}_1\right)^2}$$

Equation 12.14:

$$\mathrm{V}\left(b_1\right)=\frac{\sigma^2}{\left(1-\left(\mathrm{CORR}\left(x_{1i},x_{2i}\right)\right)^2\right)\sum_{i=1}^{n}\left(x_{1i}-\overline{x}_1\right)^2}$$

Equation 12.15:

$$s^2=\frac{\sum_{i=1}^{n}\left(y_i-\left(a+b_1x_{1i}+b_2x_{2i}\right)\right)^2}{n-3}=\frac{\sum_{i=1}^{n}e_i^2}{n-3}$$

Equation 12.16:

$$\text{SD}(b_1) = +\sqrt{\frac{s^2}{\sqrt{\sum_{i=1}^{n}(x_{1i}-\bar{x}_1)^2 - \frac{\left(\sum_{i=1}^{n}(x_{2i}-\bar{x}_2)(x_{1i}-\bar{x}_1)\right)^2}{\sum_{i=1}^{n}(x_{2i}-\bar{x}_2)^2}}}}$$

$$= +\sqrt{\frac{s^2}{\sum_{i=1}^{n}\sqrt{\left(e_{(x_1x_2)i}-\bar{e}_{(x_1x_2)}\right)^2}}}$$

Equation 12.17:

$$\text{GNI per capita} = -21{,}216 + 3{,}603(\text{CPI}) + 17.42(\text{EDI}) + e_i$$
$$(4{,}815) \quad (297.8) \quad (5.844)$$

Equation 12.18:

$$\text{child mortality} = 189.3 - 1.505\begin{pmatrix}\text{percentage of rural population with} \\ \text{access to improved water}\end{pmatrix} - .00175(\text{GNI}) + e_i$$
$$(14.95)(.2166) \quad (.0006643)$$

Equation 12.19:

$$\text{earnings} = -30{,}664 + 3{,}708.0(\text{years of schooling}) + 364.5(\text{age}) + e_i$$
$$(5{,}991) \quad (349.8) \quad (107.4)$$

Equation 12.20:

$$\left(\sum_{i=1}^{n} e_i^2\right)_{\text{U}} = \sum_{i=1}^{n}\left(y_i - (a + b_1 x_{1i} + b_2 x_{2i})\right)^2$$

Equation 12.21:

$$y_i = \alpha + \beta_1 x_{1i} + (0) x_{2i} + \varepsilon_i$$

Equation 12.22:

$$\left(\sum_{i=1}^{n} e_i^2 \right)_{\text{R}} = \sum_{i=1}^{n} \left(y_i - (a + b x_{1i}) \right)^2$$

Equation 12.23:

$$\left(\sum_{i=1}^{n} e_i^2 \right)_{\text{R}} \geq \left(\sum_{i=1}^{n} e_i^2 \right)_{\text{U}}$$

Equation 12.24:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

Equation 12.25:

$$R_{\text{U}}^2 \geq R_{\text{R}}^2$$

Equation 12.26:

$$\text{adjusted } R^2 = 1 - \frac{\left(\dfrac{\sum_{i=1}^{n} e_i^2}{n-3}\right)}{\left(\dfrac{\sum_{i=1}^{n} (y_i - \overline{y})^2}{n-1}\right)} = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n} (y_i - \overline{y})^2} \left(\frac{n-1}{n-3}\right)$$

Equation 12.27:

$$R^2 > \text{adjusted } R^2$$

Equation 12.28:

$$\frac{\left(\sum_{i=1}^{n} e_i^2\right)_{\mathrm{R}} - \left(\sum_{i=1}^{n} e_i^2\right)_{\mathrm{U}}}{j}$$

Equation 12.29:

$$\frac{\left(\dfrac{\left(\sum_{i=1}^{n} e_i^2\right)_{\mathrm{R}} - \left(\sum_{i=1}^{n} e_i^2\right)_{\mathrm{U}}}{j}\right)}{\left(\dfrac{\left(\sum_{i=1}^{n} e_i^2\right)_{\mathrm{U}}}{n-3}\right)} \sim F(j, n-3)$$

Equation 12.30:

$$F\left(1, n-3\right) = \left(t^{(n-3)}\right)^2$$

Equation 12.31:

$$H_0 : \beta_1 = 0 \text{ and } \beta_2 = 0$$

Equation 12.32:

$$y_i = a + e_i$$

Equation 12.33:

$$a = \overline{y}$$

Equation 12.34:

$$\left(\sum_{i=1}^{n} e_i^2\right)_{\mathrm{R}} = \sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2$$

Equation 12.35:

$$\frac{\dfrac{\sum_{i=1}^{n}\left(y_i-\overline{y}\right)^2 \left(1-\left(\dfrac{\left(\sum_{i=1}^{n} e_i^2\right)_{\mathrm{U}}}{\sum_{i=1}^{n}\left(y_i-\overline{y}\right)^2}\right)\right)}{\sum_{i=1}^{n}\left(y_i-\overline{y}\right)^2}}{\left(\dfrac{\left(\sum_{i=1}^{n} e_i^2\right)_{\mathrm{U}}}{\sum_{i=1}^{n}\left(y_i-\overline{y}\right)^2}\right)} \frac{n-3}{2} = \frac{R_{\mathrm{U}}^2}{1-R_{\mathrm{U}}^2}\frac{n-3}{2}$$

Equation 12.36:

$$\frac{R_U^2}{1-R_U^2}\frac{n-3}{2}=\frac{\left(\frac{b_1}{SD(b_1)}\right)^2+\left(\frac{b_2}{SD(b_2)}\right)^2-2\left(\frac{b_1}{SD(b_1)}\frac{b_2}{SD(b_2)}\text{CORR}(b_1,b_2)\right)}{1-\text{CORR}(b_1,b_2)}$$

Equation 12.37:

$$\frac{R_U^2}{1-R_U^2}\frac{n-3}{2}=\left(\frac{b_1}{SD(b_1)}\right)^2+\left(\frac{b_2}{SD(b_2)}\right)^2.$$

Equation 12.38:

$$H_0:\beta_1=\beta_2$$

Equation 12.39:

$$y_i=\alpha+\beta_1x_{1i}+\beta_1x_{2i}+\varepsilon_i=\alpha+\beta_1\left(x_{1i}+x_{2i}\right)+\varepsilon_i$$

Equation 12.40:

$$y_i=a+b\left(x_{1i}+x_{2i}\right)+e_i$$

Equation 12.41:

$$\underset{(30.67)}{\text{rent}=508.1}+\underset{(16.98)}{31.00}(\text{other rooms})+\underset{(13.90)}{115.7}(\text{bedrooms})+e_i$$

Equation 12.42:

$$\text{rent} = \underset{(39.03)}{475.9} + \underset{(7.461)}{78.89}(\text{all rooms}) + e_i$$

Equation 12.43:

$$\frac{\left(\frac{\left(\sum_{i=1}^{n} e_i^2\right)_{\text{R}} - \left(\sum_{i=1}^{n} e_i^2\right)_{\text{U}}}{j}\right)}{\left(\frac{\left(\sum_{i=1}^{n} e_i^2\right)_{\text{U}}}{n-3}\right)} = \frac{\left(\frac{153{,}975{,}279 - 152{,}470{,}299}{1}\right)}{\left(\frac{152{,}470{,}299}{997}\right)} = 9.841 \sim F(1{,}997)$$

Equation 12.44:

$$e_i^2 = c + d_1 x_{1i} + f_1 x_{1i}^2 + d_2 x_{2i} + f_2 x_{2i}^2 + g x_{1i} x_{2i} + \text{error}$$

Equation 12.45:

$$x_{1i} = c + dz_i + f_i = \hat{x}_{1i} + f_i$$

Equation 12.46:

$$\begin{aligned} y_i &= a + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i \\ &= a + \beta_1 \left(\hat{x}_{1i} + f_i\right) + \beta_2 x_{2i} + \varepsilon_i \\ &= a + \beta_1 \hat{x}_{1i} + \beta_2 x_{2i} + \left(\varepsilon_i + \beta_1 f_i\right) \end{aligned}$$

Equation 12.47:

$$x_{1i} = c + dz_i + gx_{2i} + f_i$$

Equation 12.48:

$$y_i = a + b_1\hat{x}_{1i} + b_2\hat{x}_{2i} + \text{error}$$

Equation 12.49:

$$\text{CORR}\left(\hat{x}_{1i}, \hat{x}_{2i}\right) = \text{CORR}\left(z_i, z_i\right) = 1$$

Equation 12.50:

$$\hat{x}_{1i} = c_2 + d_2 z_{1i} + g_2 z_{2i}$$

Equation 12.51:

$$\hat{x}_{2i} = c_2 + d_2 z_{1i} + g_2 z_{2i}$$

Equation 12.52:

$$\text{CORR}\left(\hat{x}_{1i}, \hat{x}_{2i}\right) \neq 1$$